NASA TT F-11,492

NOTES ON MY PAPER 'LINEAR DISCONTINUOUS BOUNDARY PROBLEMS IN THE THEORY OF FUNCTIONS, SINGULAR INTEGRAL EQUATIONS AND SOME OF THEIR APPLICATIONS'

B. V. Khvedelidze

Translation of "Zamechaniye k Moyey Rabote 'Lineynyye Razryvnyye Granichnyye Zadachi Teorii Funktsiy, Singulyarnyye Integral'nyye Uravneniya i Nekotoryye Ikh Prilozheniya'"
Soobshcheniye Akademii Nauk Gruzinskoy SSR, Vol. 21, No. 2, pp. 129-130, 1958

N68-16006 (ACCESSION NUMBER)
32 (PAGES)
(NASA CR OR TMX OR AD NUMBER)
(THRU) 1
(CODE) 19
(CATEGORY)
FACILITY FORM 602

GPO PRICE $ _____
CFSTI PRICE(S) $ _____
Hard copy (HC) 3.00
Microfiche (MF) .65
ff 653 July 65

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 JANUARY 1968

NOTES ON MY PAPER "LINEAR DISCONTINUOUS BOUNDARY PROBLEMS IN THE THEORY OF FUNCTIONS, SINGULAR INTEGRAL EQUATIONS AND SOME OF THEIR APPLICATIONS"

B. V. Khvedelidze

ABSTRACT. It is shown that in an earlier work, in which the author studied the boundary problem in the theory of functions of a complex variable, the results were incomplete. The class $D(\Gamma)$ in these results can be replaced by a class of continuous functions (class $C(\Gamma)$ by showing that if Γ is a single, closed, simple Lyapunov line and $a(t)$ is a continuous, nonzero function defined on this line, it can be represented by a canonical function. This result makes it possible to generalize the results of the earlier work.

In work [1], mentioned in the title of this note, we studied the boundary /129[1] problem of the theory of functions of a complex variable[2]

$$\Phi^+(t) = a(t)\,\Phi^-(t) + b(t), \quad t \in \Gamma, \tag{1}$$

in the case when $b(t)$ belongs to a certain class of integrable functions, $\Phi(z)$ is representable by a Cauchy type integral, while $a(t)$ belongs, except, possibly, for a finite number of points, to the subclass of continuous functions $D(\Gamma)$.

In these results, as we will show below, class $D(\Gamma)$ can be replaced by a class of continuous functions (i.e. by class $C(\Gamma)$). For this, it is sufficient to prove that if Γ is a single, closed, simple Lyapunov line and $a(t)$ is a continuous, nonzero function defined on this line, it can be represented using a canonical function.

The finite area limited by line Γ will be represented by E^+, the infinite line by E^-.

The proof of the affirmation above is based on the following lemma:

---

[1] Numbers in the margin indicate pagination in the foreign text.
[2] We will use the definitions and symbols of work [1].

Lemma. *If function* $f(z) = u(z) + iv(z)$ *is analytic in the area* E^+ *and its real portion* $u(z)$ *is continuous in* $E^+ + \Gamma$, *then the function*

$$\exp[\pm f(z)], \quad \exp[\pm i f(z)] \tag{2}$$

belongs to the class[1] E_p *with all* $p > 0$.

Actually, in the case of the function $\exp[\pm f(z)]$ this is obvious, since

$$|\exp[\pm f(z)]| = \exp[\pm u(z)]$$

is limited in $E^+ + \Gamma$. Suppose further $z = \omega(\zeta)$ is a function conformally mapping circle $|\zeta| < 1$ in area E^+. As we know from the theory of conformal mappings, derivative $\omega'(\zeta)$ is continuous in the closed circle $|\zeta| \leqslant 1$. /130
According to one theorem of V. I. Smirnov (see [2], p. 458) under the conditions of the lemma

$$\exp[\pm i f(\omega(\zeta))] \in H_p,$$

with all $p > 0$. Therefore

$$\exp[\pm i f(\omega(\zeta))] \sqrt{\omega'(\zeta)} \in H_p,$$

from which it follows that

$$\exp[\pm i f(z)] \in H_p,$$

with all $p > 0$.

Thus, the lemma is proven. The lemma is proven analogously for the area E^-.

Let us now assume that $a(t)$ is a nonzero continuous function and $\operatorname{ind} a(t) = 0$. Then, it follows from the lemma which we have proven that the canonical function for $a(t)$ is the function

---

[1] For the definition of class E_p, see [2].

$$h(\zeta) = \exp\left\{ \frac{1}{2\pi i} \int_{\Gamma} \frac{\ln a(t)}{t-\zeta}\, dt \right\}.$$

If inda(t) = κ, where κ is a nonzero integer, then

$$a(t) = a_1(t) \cdot (t - \zeta_0)^{\kappa},$$

where

$$a_1(t) = a(t)\,(t-\zeta_0)^{-\kappa},\ \zeta_0 \in E^+$$

and, consequently, $\text{inda}_1 = 0$. Thus, $a_1(t)$ is represented using the canonical function. Clearly, function $(t - z_0)^{\kappa}$ can be represented by the canonical function as well. Then the canonical function for

$$a_1(t) \cdot (t - \zeta_0)^{\kappa} = a(t)$$

is the product of the canonical functions of the cofactors (see [1], p. 78).

Finally, I note that the result presented in this note makes it possible for us to generalize all of the main results of our work [1] in a certain direction. These generalizations will be presented in a work to be published in *Trudy Tbiliskogo Matematicheskogo Instituta.*

REFERENCES

1. Khvedelidze, B. V., "Linear Discontinuous Boundary Problems in the Theory of Functions, Singular Integral Equations and Some of Their Applications," *Trudy Tbiliskogo Matematicheskogo Instituta imeni A. M. Razmadze*, v. 23, 1956.
2. Goluzin, G. M., *Geometricheskaya Teoriya Funktsii Kompleksnogo Peremennogo* [Geometric Theory of the Functions of a Complex Variable], Moscow, 1952.

Translated for the National Aeronautics and Space Administration under Contract No. NASw-1695 by Techtran Corporation, P. O. Box 729, Glen Burnie, Maryland 21061